Total contact length for a concrete cylindrical gear pair

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Abstract: *This paper gives a choice of one of four offered (for $\varepsilon_\beta < 1$) or three values of the lifetime (for $\varepsilon_\beta < 1$) of the gears pair in relation to damage from surface pressure, depending on the expected or desired intensity of damage of a cylindrical gears pair.*

Nomenclature:

Symbol	Description
bcal	Length of interface lines according to DIN 3990
lz	Total contact length
maxl	Maximum total length at points A and D
min l	Minimum totallength in points B and E
$\varepsilon\alpha$	Transverse contact ratio
$\varepsilon\beta$	Overlap ratio
px	Axial pitch
γmax	Angle between the line of lmax and axis b
γmin	Angle between the line oflmin and axis b

1. Introduction

In the phase of dimensioning pair teeththe value of the supporting surface has a key role, which is identified with the total contact lengthofthe pair teeth active at the moment. In the existing literature (DIN3990) it is calculated according to:

$$b_{cal} = \frac{b}{\cos\beta_b} \cdot \frac{1}{Z_\varepsilon^2} \qquad \textbf{(1)}$$

in which

for $\varepsilon_\beta \le 1{,}0$ $\quad Z_\varepsilon^2 = \frac{4-\varepsilon_\alpha}{3}(1-\varepsilon_\beta) + \frac{\varepsilon_\beta}{\varepsilon_\alpha}$ $\quad$ ***(2)***

and for $\varepsilon_\beta > 1{,}0$ $\varepsilon_\beta = 1{,}0$ wherein the square of the contact ratio factor isobtained

$$Z_\varepsilon^2 = \frac{\varepsilon_\beta}{\varepsilon_\alpha} = \frac{1}{\varepsilon_\alpha} = const = k \qquad \textbf{(3)}$$

so that by substituting ***(3)*** in ***(1)*** the value of the length of interface lines becomes

$$b_{cal} = \frac{b}{\cos\beta_b} \cdot \frac{1}{k} = \frac{b \cdot \varepsilon_\alpha}{\cos\beta_b} = k_1 \cdot b$$

which shows a straight line.

2. Maximum - *max lz* and minimum - *min lz* total contact length

Among others, the subject of my doctoral dissertation ***[3]*** was also the total (cumulative) length of the interface lines for cylindrical gears pair. As a result, i got the equations ***(1.56)*** and ***(1.57)*** in ***[3]***

$$\max l_z = [2\varepsilon_\beta - INT(\varepsilon_\beta)\cdot(2-\varepsilon_\alpha)]\cdot\frac{p_x}{\cos\beta_b} \text{ for } \quad \varepsilon_\beta - INT(\varepsilon_\beta) \le \varepsilon_\alpha - 1 \text{ and} \quad (5)$$

$$\max l_z = \{\varepsilon_\beta + (\varepsilon_\alpha - 1)\cdot[1+INT(\varepsilon_\beta)]\}\cdot\frac{p_x}{\cos\beta_b} \text{ for } \varepsilon_\beta - INT(\varepsilon_\beta) > \varepsilon_\alpha - 1 \quad (6)$$

for the maximum and ***(1.58)*** and ***(1.59)***in ***[3]***

$$\min l_z = [\varepsilon_\beta + INT(\varepsilon_\beta)\cdot(\varepsilon_\alpha - 1)]\cdot\frac{p_x}{\cos\beta_b} \; \varepsilon_\beta - INT(\varepsilon_\beta) \le 2 - \varepsilon_\alpha \text{ and}$$

(7)

$$\min l_z = \{2\varepsilon_\beta - (2-\varepsilon_\alpha)\cdot[1+INT(\varepsilon_\beta)]\}\cdot\frac{p_x}{\cos\beta_b} \text{ for } \varepsilon_\beta - INT(\varepsilon_\beta) > 2 - \varepsilon_\alpha \quad (8)$$

for the minimum value of the aggregate length of tangent lines in the cylindrical gear pairs.

For randomly choosen cylindrical gear pair with $m_n = 5\,[mm],\, z_1 = 17,\, z_2 = 35$ the calculated value of the transverse contact ratio ***εα = 1,4523564***, and the values of the overlap ratio $0 \le \varepsilon_\beta \le 2$, the calculated values of the aggregate lengthof the contact lines $\boldsymbol{b_{cal}}$ according to ***(1)***, **line** according to ***(4)***, ***maxl*** according to ***(5)*** i.e. ***(6)*** and ***minl*** according to ***(7)*** i.e. ***(8)*** are shown in the ***table 1***:

Table 1 *values calculated according to the equations* ***(1)*** *to* ***(8)***

εβ	b [mm]	bcal [mm] (1)	īправа [mm] (4)	max l [mm] (5) или (6)	min l [mm] (7) или (8)
0,000000	0,000000	0,000000	0,000000	0,000000	0,000000
0,200000	8,464397	11,053403	13,116960	18,063003	9,031501
0,400000	16,928794	23,011860	26,233919	36,126006	18,063003
0,452356	19,144624	26,305839	29,667707	40,854580	20,427290
0,547644	23,177362	32,487605	35,917091	45,157507	24,730217
0,600000	25,393191	35,991277	39,350879	47,521795	29,458792
0,800000	33,857589	50,128231	52,467838	56,553296	47,521795
1,000000	42,321986	65,584798	65,584798	65,584798	65,584798
1,200000	50,786383	78,701757	78,701757	83,647801	74,616299
1,400000	59,250780	91,818717	91,818717	101,710804	83,647801
1,452356	61,466609	95,252505	95,252505	106,439378	86,012088
1,547644	65,499348	101,501888	101,501888	110,742305	90,315015
1,600000	67,715177	104,935676	104,935676	113,106592	95,043589
1,800000	76,179574	118,052636	118,052636	122,138094	113,106592
2,000000	84,643971	131,169595	131,169595	131,169595	131,169595

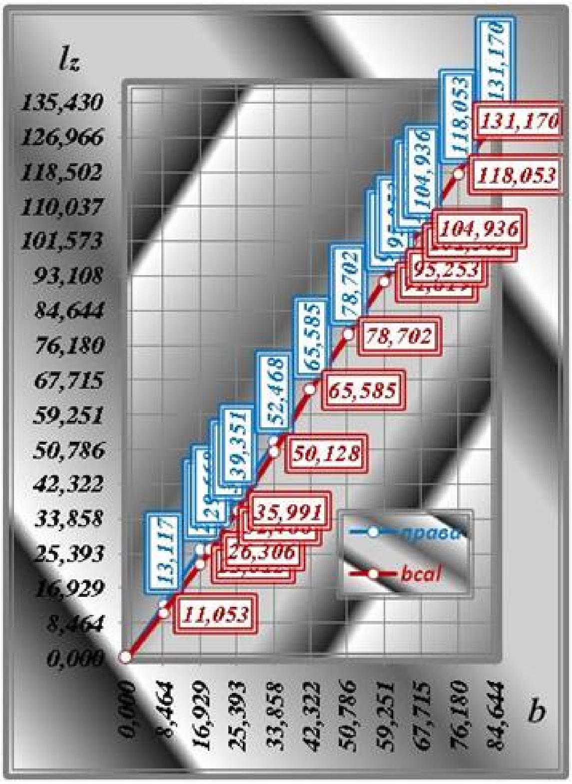

Fig.1 *Length of interface lines* ***bcal*** *by* ***(1)*** *and* ***(4)***

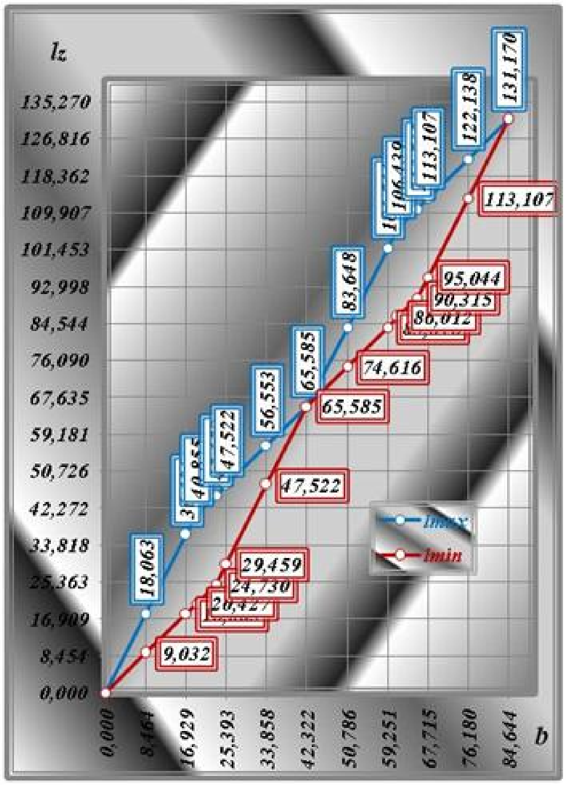

Fig.2 *Maximum* ***max l*** *according to* ***(5)*** *ie.* ***(6)*** *and the minimum* ***min l*** *length according to* ***(7)*** *i.e.* ***(8)***

According to ***Fig.1***, the values of the length of interface lines ***bcal*** according to ***DIN 3990*** to $\varepsilon_\beta \le 1$ form a curve, while for $\varepsilon_\beta > 1$ a straight line.

According to the expressions ***(5)*** or ***(6)*** and in line with ***(7)*** or ***(8)*** for the same values of the overlap ratio ε_β,, ***Fig.2*** shows the maximum ***-max.l*** and minimum ***-min.l*** value of the total length of interface lines.

To get a better view of the inter-relationship and position, all these data are displayed on ***Fig.3*** and only for the values of the aggregate (total) length of interface lines ***lz*** for integer values of the overlap ratio (***ε_β= 0;, 1.0 and 2.0)***, as well as for its values when the law to calculate the value of the maximum or minimum length of the contact lines of all coupled pairs teeth changes.

Instead of the overlap ratio ε_β, the abscissa has corresponding values of the width of the gear pair - ***b [mm]*** (see ***table 1***).

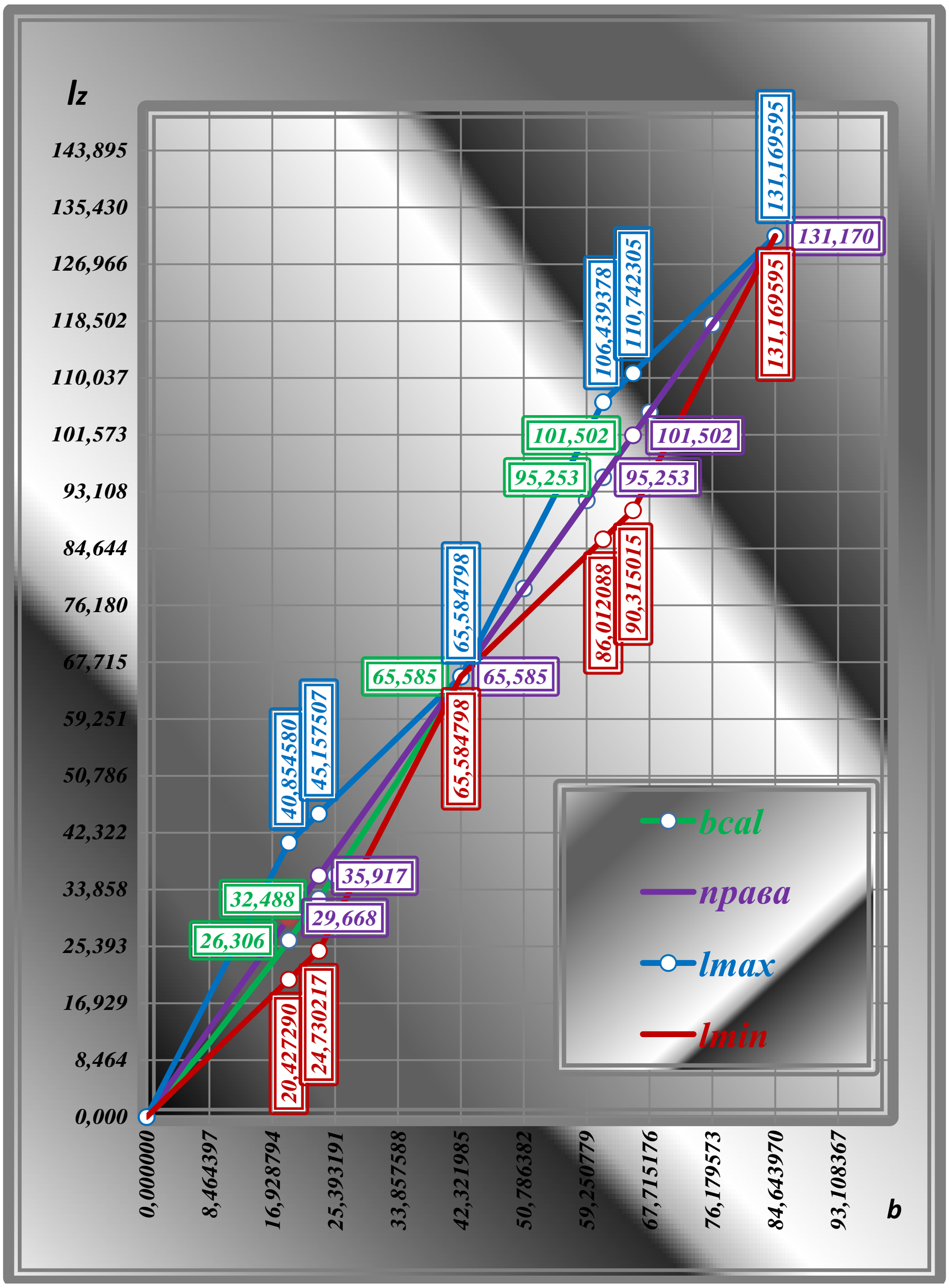

Fig.3 *The graphical representation of the total contact length according to* ***(1)****,* ***(4)****,* ***(5)*** *or* ***(6)*** *and* ***(7)*** *or* ***(8)*** *for randomly selected gear pair*

3. Analysis of the results

If we take a look at ***Fig.3*** we will notice that all four calculation procedures show that for each value of the overlap ratio, the value of the aggregate length of tangent lines is the same (ε_β ***= 1,0; 2,0; 3 0***, etc.). Therefore, we recommend integer values of overlap ratio, usually ε_β ***= 1,0***, where the width of the gears is equal to the axial pitch.

Also you will notice that all values of the length of interface lines according to **DIN 3990** are sole and placed between the maximum **max.l** and minimum value **min.l** calculated by the expressions **(5)** to **(8)**. Accordingly, the total contact length alongside the coupling of pears is not constant, but is changing from the maximum **max.l**, for coupling at the starting point **A** and point **D**, when coupling is done with ***n*** pears of theet, to minimum value **min.l** for coupling at ***B*** or exit point of the coupling ***E***, when coupling is done with ***n-1*** pears of theet.

4. Angles γ_{max} and γ_{min} against the abscissa b

The angle that is built by the starting line of the maximum total interface length ***max.l*** with axis ***b*** is γ_{max}, while the one of the minimum length of the interface ***min.l*** is γ_{min}.

For each cylinder gears pair with defined angle of tilt of teeth β, the value of these angles is constant and it is not influenced by the transmission ratio ***i***. For more tilt angles of the teeth, the value is displayed in the ***Tab. 2*** and ***Fig.4***.

Thus in the field $0<\varepsilon_\beta \leq 2-\varepsilon_\alpha$ the angle of the minimum total contact length line- ***min.l*** with abscissa – ***b*** is γ_{min}, in the field $2-\varepsilon_\alpha<\varepsilon_\beta \leq 1$ it is γ_{max}, so again γ_{min} in $1<\varepsilon_\beta \leq 3-\varepsilon_\alpha$ and again γ_{max} in $3-\varepsilon_\alpha<\varepsilon_\beta \leq 2$ etc. Inversely, the angle of the maximum total contact length line -***max.l*** with abscissa is γ_{max} in the field $0<\varepsilon_\beta \leq \varepsilon_\alpha - 1$, then γ_{min} in the field $\varepsilon_\alpha - 1<\varepsilon_\beta \leq 1$, γ_{max} in the field $1<\varepsilon_\beta \leq \varepsilon_\alpha$, again γ_{min} in the field $\varepsilon_\alpha<\varepsilon_\beta \leq 2$ etc.

Tab.2 *Angles* γ_{max} *and* γ_{min} *for angle* β

β	γ max°	γ min°
0,000	63,435	45,000
5,000	63,512	45,096
10,000	63,743	45,387
15,000	64,127	45,873
20,000	64,664	46,561
21,787	64,892	46,857
25,000	65,351	47,455
30,000	66,185	48,564
35,000	67,161	49,892
40,000	68,274	51,446
45,000	69,511	53,228

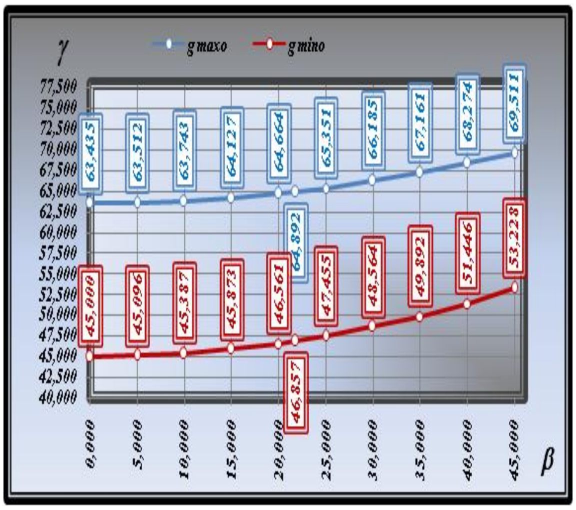

Fig.4 *Angles* γ_{max} *and* γ_{min} *depending on* β

The following ***Fig.5*** shows maximal - ***max.l*** and the minimum total contact length lines ***min.l***, indicating that the angles γ_{max} and γ_{min} remain the same, i.e. γ_{max}= ***64,892°*** and γ_{min}= ***46,857°*** forrandomly selected cylindrical gears pair with β***=21,786789°*** and several different transmission ratios.

This applies to all the other cylindrical gears pair with other values of the angle of tilt of the side lines β, whereby the value of the angles γ_{max} and γ_{min} is that shown in ***Tab.2***.

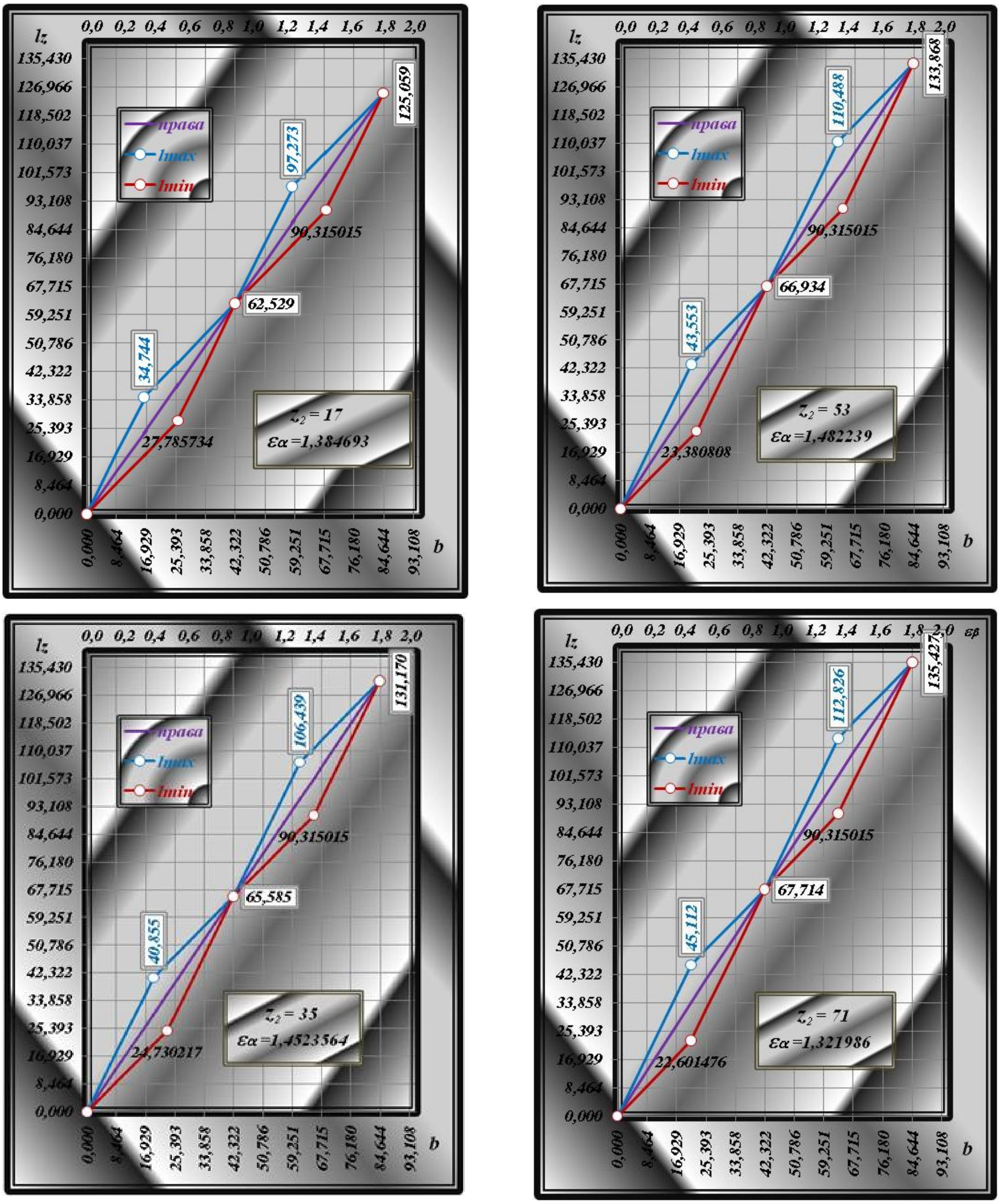

Fig.5 *Angles* $\boldsymbol{\gamma_{max}}$ *and* $\boldsymbol{\gamma_{min}}$ *for several transmission ratios of selected cylindrical geasr pair with* $\boldsymbol{\beta = 21{,}786789^o}$

5. Conclusion

1. $\varepsilon_\beta \neq n$ for n =1, 2, 3, 4 etc.

For $\varepsilon_\beta \leq 1{,}0$ there are four possible, and for $\varepsilon_\beta > 1{,}0$ three satisfactory values of the desired service life of the gear pair, as follows:

- the minimum aggregate length ***min.l*** when the damage (Pitting) at ***B*** and ***E*** is in its first phase, there is no damage in ***A*** and ***D***,
- Aggregate Length of interface lines according to ***DIN 3990*** (only where $\varepsilon_\beta \leq 1{,}0$, there is detectable damage in ***B*** and ***E***, but no damage in ***A*** and ***D***)
- the value calculated according to ***line(4)***, (the damage at ***B*** and ***E*** has increased, and there is no damage at points ***A*** and ***D***),
- the maximum aggregate length ***max. l***, (the damage in ***B*** and ***E*** it is already advanced, and with the initial occurrence of surface damage in points ***A*** and ***D***),

2. $\boldsymbol{\varepsilon_\beta = n}$ at ***n =1, 2, 3, 4*** etc.

In this case the value of the accrued total contact lines length is the same regardless the method. The initial surface damage occurs at all points of coupling and simultaneously with the expiration of the estimated useful life of the gears pair. Therefore, this option is most recommended.

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